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ADVANCED SENSITIVITY CALIBRATION OF THE LOS ANGELES STRONG MOTION ARRAY

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SUMMARY

Results are presented of recent sensitivity calibration of 76 accelerographs (SMA-1) of the Los Angeles Strong Motion Array. These have pendulum-like transducers and optical recording system. One characteristic of their design is off-axis sensitivity, which is magnified by transducer misalignment. A new calibration procedure was applied, which considers off-axis sensitivity and measures the angles of misalignment (φ and ψ), as well as the incident angle of the light beam onto the film (θ_0). These are required (1) for accurate estimation of sensitivity, and (2) for proper instrument correction of recorded accelerograms which considers also cross-axis sensitivity and misalignment. These effects are important near large acceleration peaks (approaching and exceeding $1g$), e.g. like the ones recorded near the source of the 1994 Northridge earthquake ($M_L = 6{\cdot}4$). This earthquake was recorded by 65 stations of the Los Angeles Strong Motion Array, at epicentral distances from 2 to 85 km. Histograms showing distribution of the misalignment angles, light beam incidence angle θ_0 (for unloaded position) and the transducer sensitivities are presented. These indicate that the misalignment angles are typically 1–1·5°, but may also be 3–4°. Angle θ_0 (usually neglected), is mostly between $\pm 8°$, but may reach $\pm 12°$. Assuming $\theta_0 = 0$ leads to systematically smaller values of the measured sensitivity (e.g. by $\sim 3\%$ for $\theta_0 = 8°$ and $\sim 4\%$ for $\theta_0 = 12°$). Comparison of the newly measured sensitivities with those measured prior to installation (in 1979/1980), s^{old}, shows that, in general, the new values are systematically smaller. The difference is typically within 5 per cent, but in some cases is as large as 10 per cent. Other principal sources of the observed differences and their mechanisms are discussed. Those include long-term changes in the transducers (e.g. change of stiffness, reflected in changes of the natural frequency) and differences in the calibration procedure (e.g. errors associated with manual reading film records with tilt test data, and with transducer and instrument housing misalignment). The presented results may be considered typical of similar strong motion arrays worldwide. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: instrumentation; accelerograph calibration; strong motion data; strong motion recording; Los Angeles strong motion array; Northridge earthquake

INTRODUCTION

The Los Angeles Strong Motion Array[1] consists of 79 stations (all recording ground motion at 'free field' sites), and has been in operation since 1979/1980. One of the most important earthquakes which shook the Los Angeles metropolitan area is the $M_L = 6{\cdot}4$ Northridge earthquake of January 17, 1994. It was recorded by 65 stations of this array,[1] many close to the earthquake source (25 of the stations were within 30 km radius from the epicenter). The recorded peak accelerations approached $1g$ at several of the stations. Many of the recordings are being used to interpret damage to structures, ground movement and non-linear soil response,

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and it is important that the recorded data is processed accurately. Therefore, all the instruments were recalibrated using an advanced static calibration test. This paper describes the new calibration results, compares the new and the old transducer sensitivity values and discusses the differences.

The recording instruments are SMA-1 accelerographs, having pendulum like transducers and optical recording system.[2,3] The optical system achieves amplification factor of 4 by reflecting a light beam, twice from a mirror attached to the transducer pendulum. The light beam is then projected onto a 70 mm wide film moving to a nominal speed of 1 cm/s; the optical arm is 125 mm. Nominal values for the transducer natural frequency, damping and sensitivity are 25 Hz, 0·6 of critical and 1·7 cm/g.

One characteristic of accelerographs with pendulum-like transducers is cross-axis sensitivity, referring to a transducer being sensitive to and recording motions off its nominal sensitivity axis. The associated effects are further magnified if the transducer penduli are not perfectly aligned to their nominal orientation (longitudinal, transverse and vertical axes of the instrument housing). A consequence of this is that the nominal sensitivity axes of the three transducers are not mutually perpendicular. The resulting errors in the measured motion are significant only around large peak accelerations,[2,4] close to and exceeding gravity ($1g$). For SMA-1 accelerographs, acceleration of $1g$ results in $\sim 2^\circ$ deflection of the pendulum, further amplified to $\sim 8^\circ$ by the optical system, and resulting in $\sim 1{\cdot}7$ cm deflection of the trace on the film. Therefore, misalignment of the transducer penduli by few degrees is of same order of magnitude as the angular response of the penduli to $1g$ acceleration. Considering cross-axis sensitivity and misalignment is important (1) to evaluate accurately the transducer sensitivity and also (2) to reconstruct accurately three orthogonal components of motion from the recording. The latter can be done by performing appropriate instrument correction.[2,4]

Misalignment and cross-axis sensitivity of accelerographs with pendulum-like transducers were first analysed by Skinner and Stephenson,[5] followed by Wong and Trifunac.[4] Recently, Todorovska *et al.*[2] revisited this problem, motivated by the large accelerations recorded during the 1994 Northridge, California, earthquake.[1,6] They developed an algorithm for routine processing of field test measurements (using a specially designed tilt table) and for the first time presented statistics for the values of the angles of misalignment for a strong motion array. They also proposed two instrument correction algorithms which correct for the associated effects, and analysed possible implications on selected accelerograms of the Northridge earthquake. Besides misalignment and cross-axis sensitivity, their procedure also considers and corrects for inclined light beam incidence onto the film. The mathematical model and the proposed algorithm are also described in Reference 3. This paper summarizes results of 91 tests performed on 76 accelerographs of the Los Angeles Strong Motion Array (some instruments were tested twice, prior and after adjustments of the transducer alignment). The calibration was performed within a period of 6 months following the earthquake.

The purpose of this paper is (1) to encourage calibration of strong motion accelerographs following recording of large amplitude strong ground motion, and (2) to present the improvements in sensitivity calibration which result mainly from the completeness of the mechanical model of the recording transducer. The calibration method used in this work involves static testing only. However, the test results (sensitivity constants and misalignment angles) are used in subsequent accelerogram data processing and contribute to correction at all frequencies of the recorded motion. The transducer natural frequency and damping, which define its behaviour mainly at high frequencies (>20 Hz) do not interact dynamically with the sensitivity estimates, and therefore are not the subject of this study.

The Los Angeles Strong Motion Array is a typical array of SMA-1 accelerographs. Therefore, the calibration results presented in this paper can be considered as representative for similar arrays worldwide, and can be used to infer approximately the accuracy of calibration and data processing of many recordings by analogue accelerographs. Also, the new calibration results and accelerogram correction algorithms[2] can be used to reprocess the recordings of the Northridge earthquake at stations of the Los Angeles Strong Motion Array and evaluate the consequences on various characteristics of the processed data. This task is outside the scope of this paper.

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METHODOLOGY

Static tilt test

The calibration test is as described in References 2 and 3. In brief, the instrument is tilted in the field (through 30°–90°) by using a calibrated tilt table. Rotation through $\pm 30°$ imposes $\pm 0{\cdot}5g$ acceleration on the horizontal transducer and $-0{\cdot}134g$ for the vertical transducer. Rotation through $\pm 90°$ imposes $\pm 1g$ acceleration for the horizontal transducers and $-1g$ for the vertical transducer. The film record is then developed, digitized using a flat bed scanner (at 300 or 600 points/inch scanning resolution), and the digital traces are processed by the computer program TILT.[2,3] The static equilibrium equations for the various tilted positions of the instrument form an overdetermined system of equations which is solved by least squares. The output consists of the angular amplification of the pendulum, a^w (in units g/rad), two misalignment angles for each transducer (angles φ and ψ, see Figure 1) and the angle of the light beam incident onto the film, θ_0 (see Figure 2).

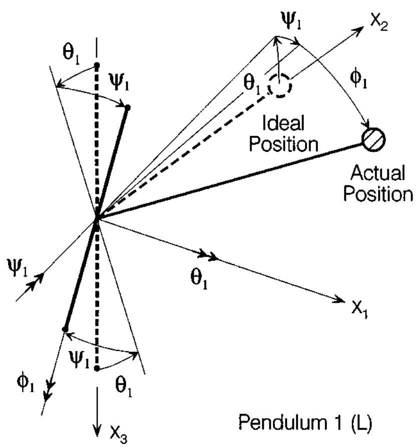

Figure 1. Misalignment angles for Pendulum 1 (L) shown by arcs and by double arrows

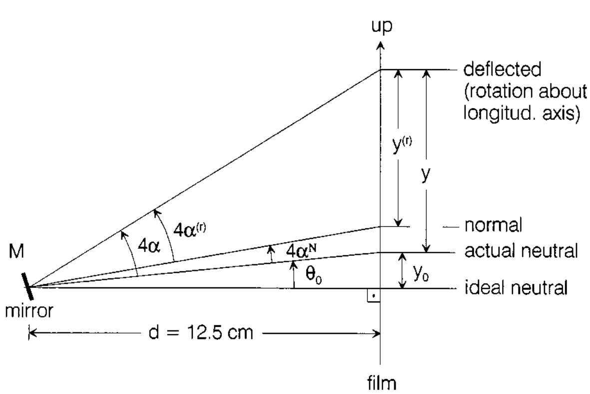

Figure 2. Unfolded ray path between the last reflection from the mirror attached to and moving with the transducer pendulum and the film plane. θ_0 is the incident angle of the ray onto the film when the pendulum is in neutral position

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In the mathematical model for the accelerograph,[2,3] three angles of misalignment are defined, φ, ψ and θ. Figure 1 shows schematically the model (pendulum) for the L transducer of an SMA-1 accelerograph. The pivot axis is shown by the line segment with circles at the ends. The pendulum is perfectly aligned if the pivot axis is parallel to the X_3-axis and the pendulum is parallel to the X_2-axis. The nominal sensitivity axis is parallel to the X_1-axis. Three consecutive rotations, through angles φ, ψ and θ, would bring the pendulum from the ideal to the actual (misaligned) position. These angles are shown in Figure 1 by arcs and also by double arrows. Angle φ represents rotation about the pivot axis, while angles ψ and θ represent rotations of the pivot axis. An analysis of the model shows that angle θ appears in the equations of motion, as well as in the static equilibrium equations of the model in terms of order 2 or greater, which drop out if the equations are linearized. The algorithm used here[2,3] is based on the linearized equations, and therefore solves only for angles φ and ψ.

The relationship between the response of the pendulum (angle α) and the trace deflection on the film, y, is illustrated in Figure 2, which shows unfolded path of the ray from the last (second) reflection from the mirror attached to the transducer to a cylindrical lens focusing the ray onto the film. The distance between the mirror and the focusing lens is $d \approx 12{\cdot}5$ cm. Ideally, when no load is applied, the light beam should be perpendicular to the film, and this position is called 'ideal neutral'. Due to design constraints, need to position the acceleration traces properly onto the film, or improper setting, the 'actual neutral' is off the 'ideal neutral' position, at distance y_0, and the ray incident angle is θ_0. The 'normal' position indicates the trace position when the instrument is not tilted and when it is not moving. In general, it is different from the 'actual neutral' position, due to misalignment and also if the accelerograph is not perfectly horizontal. The calibration algorithm evaluates the actual position and corrects for the associated effects.[2,3] The absolute deflection angle of the pendulum, α, and trace deflection, y, by definition are measured from the 'actual neutral' position. However, what can be measured is the trace deflection relative to the 'normal' position, $y^{(r)}$. Angles $\alpha^{(r)}$ can be calculated from the trace deflections $y^{(r)}$, given the ray incident angle, θ_0, from the geometry of the ray paths in Figure 2.

If $\theta_0 = 0$ (the 'actual neutral' coincides with the 'ideal neutral' position), then the deflection on the film, $y^{(r)}$, from positive and negative 'loads' (e.g. $+1g$ and $-1g$) would be symmetric. Angle $\theta_0 \neq 0$ can be evaluated by iteration,[2] using deflections $y^{(r)}$ for 'loads' $+1g$ and $-1g$. Angle θ_0 is in general non-zero.

Accuracy

The accuracy of the calibration procedure is determined by the resolution of the scanner, which is used to convert the film image of the test results into digital form, by the quality of the traces recorded on the film, and by the completeness of the calibration process. For 300 points/in resolution (1 in $= 2{\cdot}54$ cm), the distances between digitized points can be resolved within $2{\cdot}54/300 = 0{\cdot}0085$ cm. (This corresponds to light beam rotation angle $4\alpha^{(j)} = (0{\cdot}0085/12{\cdot}5) \times (180/\pi) = 0{\cdot}039^{\circ}$, and to pendulum rotation angle $\alpha^{(j)} = 0{\cdot}039/4 \approx 0{\cdot}01^{\circ}$ (see Figure 2).) The actual working accuracy of the algorithm is however much higher than 0·0085 cm and 0·01°, because the digitized trace position is averaged over 1–2 cm (1–2 s) length for each instrument position of the tilt test. For 300 points per inch scanning resolution (118 points per cm), there are at least 200 points (measurements of the trace position) in a trace segment. The accuracy thus depends mainly on the quality and width of the recorded trace. A study of the stability and of the accuracy of the algorithm for processing the test results[2] showed that twice higher scanner resolution (i.e. 600 versus 300 dots per inch) does not lead to twice higher accuracy, due to 'noise' in the measurements resulting from e.g. film distortion (partly eliminated by use of a fixed baseline recorded on the film), and the finite trace thickness and trace quality. This 'noise' contributes to and is reflected in the standard deviation of the least-squares estimates of the unknowns.

The accuracy of reading manually the trace deflection from the film is about 0·01 cm, not too different from 0·0085 cm, which is the accuracy associated with scanner resolution of 300 dots per inch. However, the difference in error of reading the trace deflection is a factor of 10 in favour of the procedure involving

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digitization, because the manual reading is performed once or twice, while the 'reading' via digitization is performed at least about 200 times.

RESULTS

Ninety-one tests were performed on 76 instruments of the Los Angeles Strong Motion Array. In what follows, typical results are presented. A more complete set, including additional results based on various simplifying assumptions in the mathematical model, can be found in Reference 2. These assumptions are, e.g., neglecting correction for inclined ray incidence on the film (assuming $\theta_0 = 0$), and neglecting correction for tilt table which is not perfectly horizontal during the tilt test.

Statistics

Figures 3 and 4 show distribution of the measured misalignment angles φ and ψ. It is seen that angles φ are distributed over a large interval (between $-4°$ and $4°$) than angles ψ (mainly between $-1°$ and $1°$). The distribution for φ for the L transducer is biased towards negative values, while for the T and V transducers, the distributions are approximately symmetric. The distributions of ψ are biased towards positive values for all the three transducers.

The output of the new algorithm for processing the field test results is the angular amplification constant, a^w, in units g/rad. 'Standard' calibration procedures measure the sensitivity $s^{y,w}$, in cm/g, directly from trace deflections when 'loads' of $\pm g$ or $\pm \frac{1}{2} g$ are applied. While a^w is a constant for the transducer, from Figure 2 it is seen that the trace deflection $y^{(r)}$ is not directly proportional to the applied acceleration, but is a transcendental function of the acceleration. To express the newly calculated transducer sensitivity in familiar units, we define sensitivity $s^{y,w}$ as

$$s^{y,w} = d \tan(4/a^w) \tag{1}$$

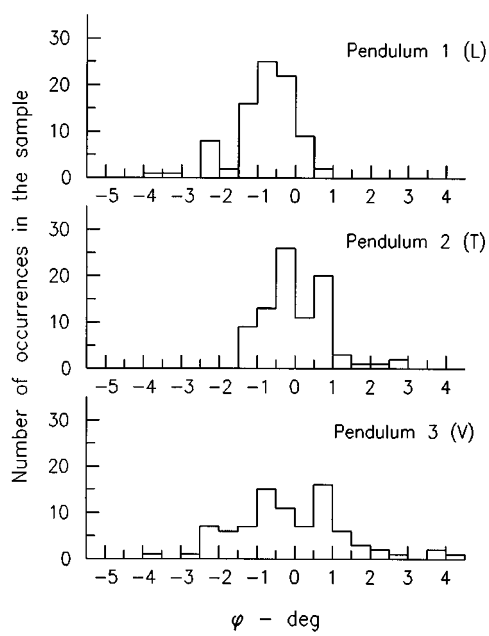

Figure 3. Histogram of values of misalignment angle φ for the Los Angeles Strong Motion Array. For Penduli 1 and 2 (L and T), $\theta_0 \neq 0$ was considered, while for Pendulum 3 (V) it was assumed that $\theta_0 = 0$

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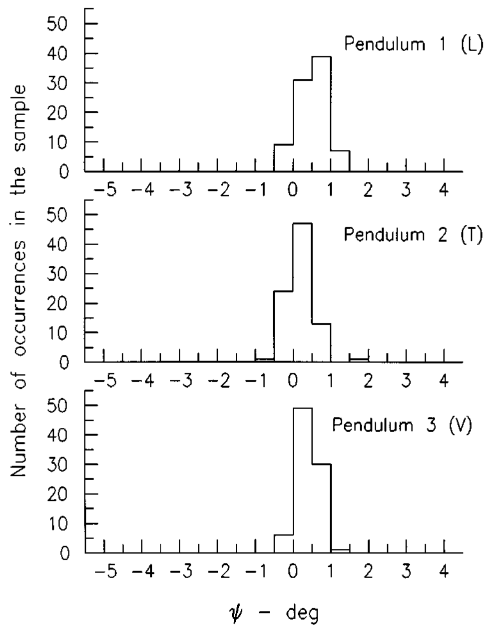

Figure 4. Histogram of values of misalignment angle ψ for the Los Angeles Strong Motion Array. For Penduli 1 and 2 (L and T), $\theta_0 \neq 0$ was considered, while for Pendulum 3 (V) it was assumed that $\theta_0 = 0$

which would be the trace deflection from acceleration of $1g$ if the light beam is initially perpendicular to the film ($\theta_0 = 0$). Figures 5 and 6 show distribution of values of sensitivity $s^{y,w}$ and of the incident angle θ_0. It is seen from Figure 5 that the distribution of sensitivity is broad (between 1·45 and 2·0 cm/g). Distribution of θ_0 is shown only for the L and T transducers. The V transducer has different configuration from the other two in that the zero acceleration position corresponds to the static equilibrium position under the pendulum weight. Therefore, it requires different algorithm to evaluate the corresponding angle θ_0 and additional tests to achieve faster and sure convergence.[2] It is seen from Figure 6 that for the L and T transducers, θ_0 is distributed between -14° and 14°, but mostly between -8 and 8°.

The effect of correcting for angle θ_0 on the estimates of misalignment angles φ and ψ and sensitivity $s^{y,w}$ has been analysed in Reference 2. This analysis shows that the standard deviation of the least-squares estimates of a^w, φ and ψ is significantly reduced if the algorithm considers $\theta_0 \neq 0$. For larger values of θ_0, there are systematic differences in the estimates of misalignment angle ψ and of sensitivity. The sensitivity is overestimated if correction for θ_0 is not made, both for positive and for negative values of θ_0. The difference versus θ_0 looks like a 'parabola', with amplitudes equal to 0 per cent for $\theta_{0,i} = 0^\circ$, 1 per cent for $\theta_{0,i} \approx 5^\circ$, about 2 per cent for $\theta_{0,i} \approx 8^\circ$, and 3·5 per cent for $\theta_{0,i} \approx 10^\circ$. Therefore, correcting for inclined light beam incidence is important for obtaining accurate estimates of sensitivity, and also for accurate conversion of the trace deflection of recorded accelerograms into corresponding angle of pendulum deflection. (While the conventional data processing programs scale the trace deflection on the film directly in acceleration units using sensitivity in cm/g, and then perform instrument correction, the new instrument correction procedure, which corrects for misalignment, is in terms of the angular deflection of the pendulum[2].) Correcting for off-normal light beam incidence ($\theta_0 \neq 0$) is also important for accurate estimation of misalignment angles[2] φ and ψ.

Todorovska *et al.*[2] analysed the accuracy in the determination of the misalignment angles and sensitivity constants and the stability of the procedure. For the results presented in Figures 3–5, the standard deviation of the estimates is as follows. For most cases, σ for angle φ is not more than 0·2 to 0·3°, and for angle ψ not

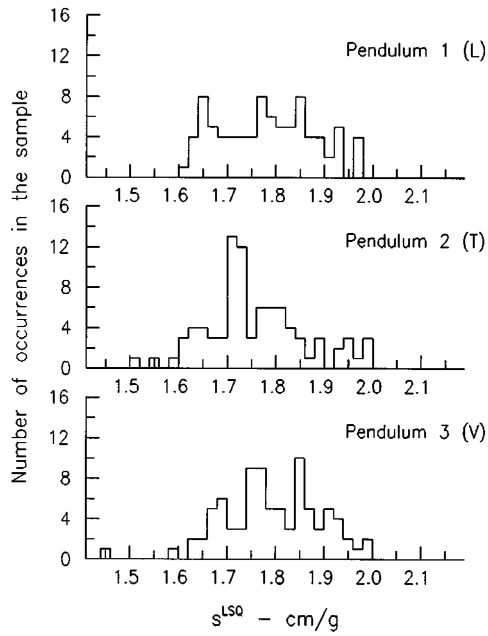

Figure 5. Histogram of values of sensitivity s^{LSQ} for the Los Angeles Strong Motion Array. For Penduli 1 and 2 (L and T), $\theta_0 \neq 0$ was considered, while for Pendulum 3 (V) it was assumed that $\theta_0 = 0$

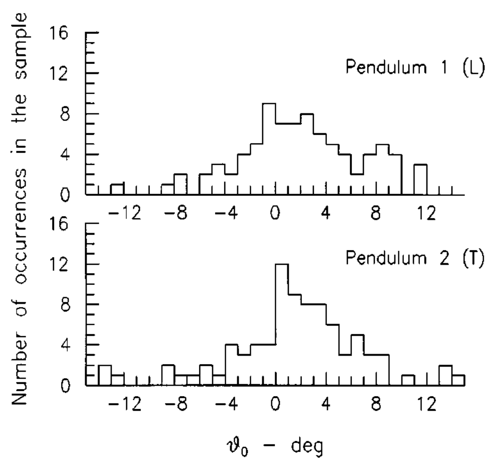

Figure 6. Histogram of incident angle θ_0 for the Los Angeles Strong Motion Network, for Penduli 1 and 2 (L and T). The accuracy of θ_0, set for the iteration loop, is 0.001°

more that 0·1°. To test the performance of the complete algorithm, several transducers were deliberately misaligned by tilting and rotating the transducer magnets (see Figure 2 in Reference 3) within the accelerograph housing, by $\Delta\psi$ in the range from 1 to 3°. The test results agreed with the independently measured misalignment angles. These tests confirmed the accuracy of the entire procedure and the above quoted accuracy of individual estimates.

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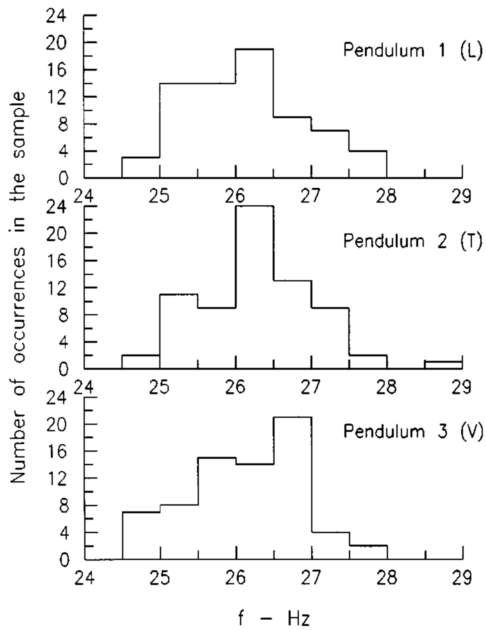

Figure 7. Histogram of values of natural frequency s^{LSQ} for the Los Angeles Strong Motion Array

Figure 7 shows distribution of the natural frequencies, f, in Hz, measured manually from the free vibration decay tests ran at the time of the static tilt test. Although the analysis in this paper focuses on results of the static tilt test, the results of the 'new' natural frequency calibration and the observed changes with respect to the 'old' values will be useful in explaining the trends of changes of the transducer sensitivity. It is seen from Figure 7 that f is distributed between 24·5 and 28 Hz.

For film speed of 1 cm/s, and $f \sim 25$ Hz, 1 cm long calibration test corresponds ~ 25 cycles. Typical resolution of reading over the length of 1 cm on the film is 0.01 cm. Then, the accuracy of estimating the natural frequency is between 1 and 1·5 per cent.

ANALYSIS AND DISCUSSION

Comparison of the new and old sensitivities

The Los Angeles Strong Motion Array was installed during the end of 1979 and the beginning of 1980. The sensitivities measured by the manufacturer, and verified at the Strong Motion Recording Laboratory at USC prior to the installation, have been used in data processing ever since. We refer to these values as s^{old}. It is interesting to see how these compare with the sensitivities evaluated 15 years later, by program TILT.

Figure 8 shows plots of s^{old} versus sensitivities s^{LSQ} (evaluated via equation (1), from the least-squares solution for the angular amplification), for the L, T and V transducers. Heavy horizontal lines show 2σ intervals for the s^{LSQ} estimate. The estimates s^{old} and s^{LSQ} are correlated, but there is significant scatter and, in general $s^{old} > s^{LSQ}$, for the L and T transducers. Figure 9 shows plot of f^{old} versus f. The observed scatter is similar as in Figure 8. It is seen that, in general, $f > f^{old}$, suggesting 'stiffening' of the transducer spring. The relationship between changes in f and in sensitivity is discussed in the next section.

In Figure 10, the normalized difference $(s^{LSQ} - s^{old})/s^{LSQ}$ has been plotted versus θ_0 for the L and T transducers, and uniformly spread along an arbitrary horizontal axis for the V transducer (θ_0 was not

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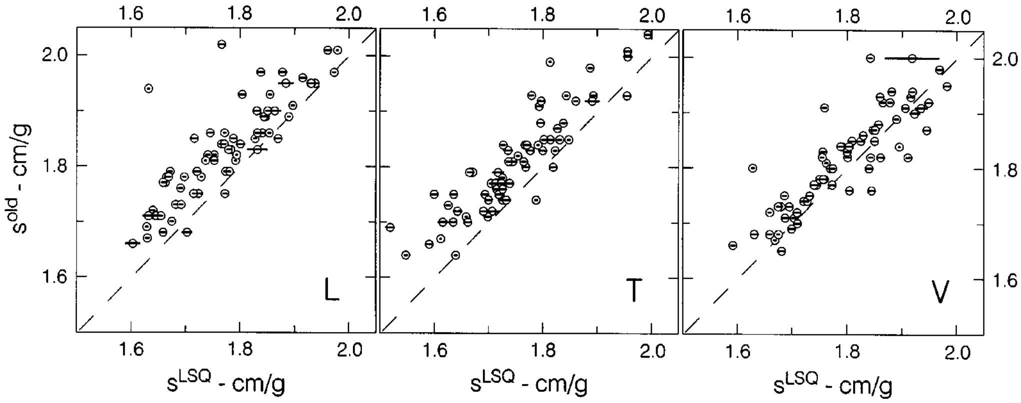

Figure 8. Comparison of the old (s^{old}) and recent (s^{LSQ}, least-squares solution, considering misalignment, cross-axis sensitivity and off-normal ray incidence) sensitivity estimates

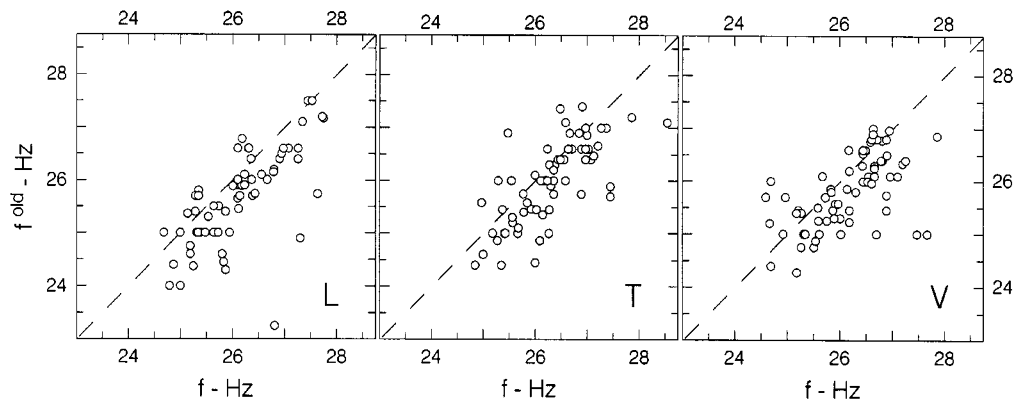

Figure 9. Comparison of the old (f^{old}) and recent (f) measurements of transducer natural frequency

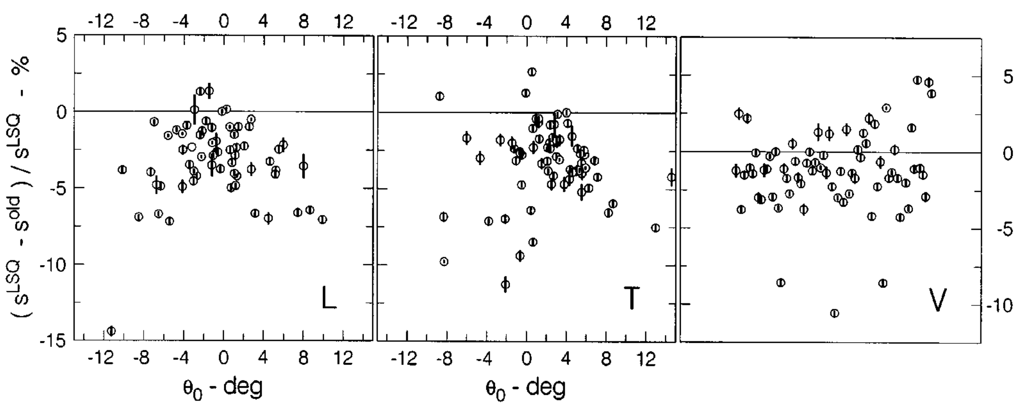

Figure 10. Percentage difference between the old (s^{old}) and recent (s^{LSQ}, least-squares solution, considering misalignment, cross-axis sensitivity and off-normal ray incidence) sensitivities versus ray incident angle, θ_0, for the L and T transducers, and versus an arbitrary axis for the V transducer

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measured for the V transducer). It can be seen that, in general, $s^{\mathrm{LSQ}} < s^{\mathrm{old}}$, and that for the L and T transducers, the points are scattered about a concave 'parabola' with maximum at $(s^{\mathrm{LSQ}} - s^{\mathrm{old}})/s^{\mathrm{LSQ}} < 0$. The average difference in sensitivity is about 2·5 per cent for $\theta_0 = 0$.

The differences between the old and new estimates of sensitivity result from differences in the calibration procedure and from long- and short-term changes in the transducers. The following explains briefly the sources and the range of the observed differences.

Change in stiffness

The change in the stiffness of the transducer spring is reflected in changes of sensitivity and of natural frequency. The changes caused by aging might be systematic (e.g. spring 'stiffening' or 'softening'), while changes due to temperature variations are expected to be more 'random' and difficult to trace. If the spring is linear, then the ratio of the stiffness $K/K^{\mathrm{old}} = (f/f^{\mathrm{old}})^2 = a^{\mathrm{LSQ}}/a^{\mathrm{old}}$, where f and f^{old} are the recently measured and the old values of the natural frequency, a^{LSQ} is the recently measured angular amplification constant (in g/rad) and a^{old} is equivalent constant evaluated from the old sensitivity (in cm/g) assuming $\theta_0 = 0$ and optical arm $d = 12{\cdot}5$ cm ($a^{\mathrm{old}} = 4/\tan^{-1}(s^{\mathrm{old}}/d)$, where s^{old} is in cm/g). Figure 11 shows plots of $a^{\mathrm{LSQ}}/a^{\mathrm{old}}$ versus $(f/f^{\mathrm{old}})^2$. (Typical accuracy with which f can be measured from decaying free vibration test on the film is better than 1·5 per cent. The accuracy of $(f/f^{\mathrm{old}})^2$ coordinates in Figure 11 is then better than about 6 per cent). If the spring is linear and the changes in sensitivity and natural frequency were only due to change of stiffness, then all the points would lie on the 45° line. It is seen that $a^{\mathrm{LSQ}}/a^{\mathrm{old}}$ and $(f/f^{\mathrm{old}})^2$ are correlated, indicating that change in stiffness of the spring contributed to the observed differences in sensitivity (Figures 8 and 9), but the apparent trend does not follow a 45° line.

Modelling and detailed interpretation of the observed differences is beyond the scope of this paper. Here, we only summarize the contributing mechanisms to the overall differences in transducer sensitivity, and suggest simple and possible causes. For the majority of the data points, $K > K^{\mathrm{old}}$; the average difference is less than 1 per cent. One simple and plausible explanation is that the change in stiffness is temperature related. The old calibration tests were performed in the laboratory, with essentially constant temperature environment (68–70°F). The recent tests were carried out in the field at higher temperatures, typical of the Los Angeles climate (April through October, 9 am to 5 pm, say 60–90°F).

The data in Figure 11 show that the points are scattered about a line with slope smaller than 45°. (Least-squares fit through the points in Figure 11 gives, respectively, for the L, T and V transducers,

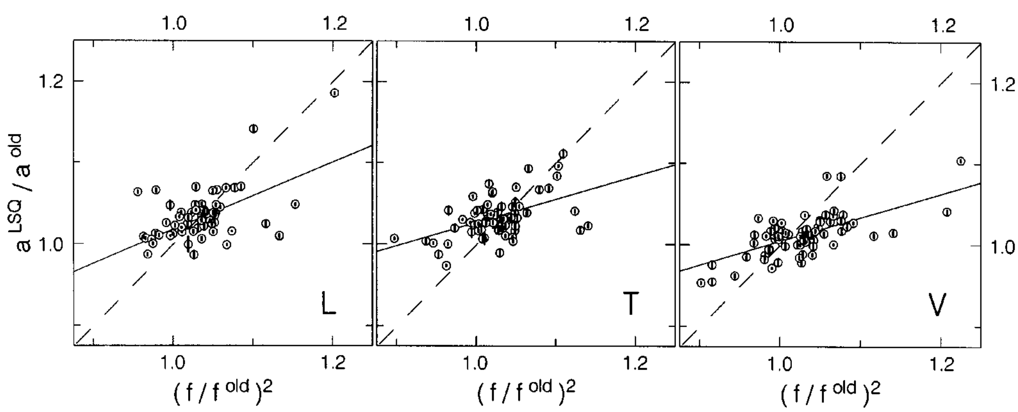

Figure 11. A scatter diagram of ratio of recent and old values of angular amplification constant ($a^{\mathrm{LSQ}}/a^{\mathrm{old}}$) and square of the ratio of the recent and old values of natural frequency ($(f/f^{\mathrm{old}})^2$)

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$y = 0{\cdot}41x + 0{\cdot}60$, $y = 0{\cdot}29x + 0{\cdot}74$ and $y = 0{\cdot}29x + 0{\cdot}71$ where $y = a^{\mathrm{LSQ}}/a^{\mathrm{old}}$ and $x = (f/f^{\mathrm{old}})^2$.) Other possible reasons for these trends might be sought in non-linear behaviour of the leaf spring under different temperature conditions (non-linear elastic or geometric non-linearity), and increased fixity and stiffness of the leaf springs caused by oxidation. At this time, we do not know the variations of the oxidation rates in time and from one station to another. A more detailed explanation for the above trends requires further research (including, repeated calibration of the same transducers, under the same conditions, during the next 5 to 10 years), and is out of the scope of this paper.

Inclined ray incidence

The 'parabolic' trend in Figure 10 is due to the effect of the angle θ_0 (Figure 2) which was not considered in the old calibration procedure. To demonstrate this, in Figure 12, the normalized difference is shown between two of the recent estimates of sensitivity, one considering all corrections (s^{LSQ}) and the other one determined 'the old way', i.e. directly from the difference between the trace deflection for $+1g$ and $-1g$ loads ($s^{(y,\,g)}$), plotted versus the light beam incident angle, θ_o. The error bars shows the 2σ interval for the s^{LSQ} estimate. The 'parabolic' trend is obvious. It is seen that ignoring θ_0 leads to larger estimates of sensitivity, up to about 5 per cent for $\theta_0 = \pm 12^\circ$, and in one case up to about 7 per cent for $\theta_0 = 14^\circ$.

Transducer and instrument housing misalignment

The procedure of measuring the old sensitivity constants ignored misalignments. The misalignment of the transducers relative to the instrument housing was measured by angles φ and ψ (Figure 1). If the instrument housing is perfectly aligned to horizontal, misalignment of $\psi = 1^\circ$ results in applying a load of $(\sin 1^\circ)(g) = 0{\cdot}0174g$, and causes trace deflection from the real neutral by 0.03 cm, and error in sensitivity of ~ 2 per cent.

The new calibration procedure corrects for angles δ_1 and δ_2 which define the position of the tilt table (i.e. of the instrument housing, securely fastened to the tilt table base plate) relative to horizontal. This correction eliminates the need to adjust the tilt table to perfect horizontal position, and speeds up the field work. Another advantage of including such a correction in the algorithm is that it includes corrections for imperfections of the instrument housing which would result in $\delta_1, \delta_2 \neq 0$ even if the tilt table base plate were

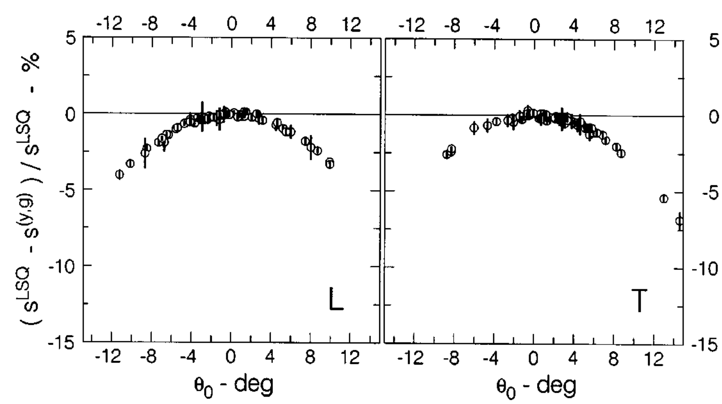

Figure 12. Percentage difference between two recent measurements of transducer sensitivity, s^{LSQ} and $s^{(y,g)}$, versus ray incident angle, θ_0, for the L and T transducers (s^{LSQ} is the least squares solution, considering misalignment, cross-axis sensitivity and off-normal ray incidence, and $s^{(y,g)}$ is the estimate obtained directly from the trace positions on the film corresponding to $+g$ and $-g$ loads)

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adjusted to be perfectly horizontal†). These angles would then represent the misalignment of the housing relative to its ideal position. These two angles of misalignment (δ_1 and δ_2) are common for all the three transducers and cannot be 'absorbed' entirely within the transducer misalignment angles (φ and ψ). Non-zero values of δ_1 and δ_2 lead to biased results of the tilt test,[2] especially for the transducer misalignment angles ψ (due to strong coupling). For the 91 tests performed on the Los Angeles Strong Motion Array, δ_1 and δ_2 were mostly within $\pm 1^\circ$, and were not larger in absolute value than $\sim 2^\circ$.

The old sensitivity calibration procedure did not correct for misalignment of the instrument housing. Misalignment of $\sim 1^\circ$ would result in error in sensitivity estimate of ~ 2 per cent for the L and T transducers (see the discussion on the error due to $\psi = 1^\circ$ in the previous paragraph). For vertical transducer the amplitude of the error in measured sensitivity (caused by φ and ψ) is comparable, but can be eliminated by averaging the results for $+30^\circ$ with -30° and $+90^\circ$ tilt angles.

Accuracy of reading the trace position

The error in manually reading the trace positions from the film was discussed in the section on the methodology of this paper. It was stated that the accuracy of the new procedure is higher than of the old procedure. An estimate for this type of error in measuring sensitivity of L and T transducers would be ~ 1 per cent, for manually reading the trace position, within 0·01 cm. The error in measuring the sensitivity of the vertical transducer for $-0{\cdot}134g$ load ($\pm 30^\circ$ tilt angle) is large, about 4 per cent, but can be reduced to about ~ 3 per cent by averaging the results for $\pm 30^\circ$ and $\pm 90^\circ$ tilt tests.

Load-dependent sensitivity

Another contributor to the differences between the old and recent estimates of sensitivity is that the sensitivity measured 'the old way', directly from the film, is a function of the load (recall that the angular sensitivity, in rad/g, is a constant, while the sensitivity in cm/g is a transcendental function of the angle of deflection, and therefore depends on the load.) To illustrate this effect, in Figure 13, two types of recent estimates of sensitivity are shown plotted one versus the other, both evaluated 'the old way' (directly from trace deflection on the film), but for different loads. On the horizontal axis, the sensitivity $s^{(y,g)}$ is shown, determined for the L and T transducers from the difference in trace deflections for $+1g$ and $-1g$ loads (trace deflections are on both sides of the normal position), and for the V transducer from the trace deflections from two $-1g$ loads[2,3] (both trace deflections are on one side of the normal position). On the vertical axis, the corresponding sensitivities evaluated from 30° tilted positions are shown, $s^{(y,g/2)}$ for the L and T transducers, and $s^{(y,0{\cdot}134g)}$ for the V transducer, i.e. for loads $\pm g/2$ for the former and for load $-0.134g$ for the latter. The trend seen in these plots is that the sensitivity estimated from the $1g$ loads is larger than the sensitivity estimated for the smaller loads. (The s^{LSQ} sensitivity plotted in Figures 8 and 10 is the apparent sensitivity in cm/g for $1g$ load and assuming $\theta_0 = 0$.) From the geometry in Figure 2, it can be shown[2] that, for perfectly aligned transducers and $\theta_0 = 0$, the difference in estimating sensitivity from $1g$ and $g/2$ trace positions is about 0·5 per cent. Figure 13 shows that the systematic difference between $s^{(y,g)}$ and $s^{(y,g/2)}$ for the L and T transducers is <1 per cent. For the V transducer, the scatter is larger, because of the smaller relative accuracy of measuring the trace deflection for $-0{\cdot}134g$ load, about 3 per cent.

Figure 13 also shows that $s^{(y,0{\cdot}134g)}$, for the V transducer, is smaller than $s^{(y,g)}$ by ~ 3 per cent. Part of this bias results from contribution of pendulum deflection angle (about 2° for $1g$ load), for pendulum at or near the normal position, and from absence of this contribution for 90° clockwise and counterclockwise tilt angles ($\sim 0g$ loads). This bias is further affected by misalignment angles φ of the vertical transducer and by

† The acceleration transducers are bolted to the base plate inside the SMA-1 accelerograph housing, and the position of the housing is determined by the precision of three legs which rest on the tilt table surface. Deviations from perfect dimensions of the base plate and from casting the instrument box contributes to additional rotations, which to a given transducer appear as δ_1 and δ_2.

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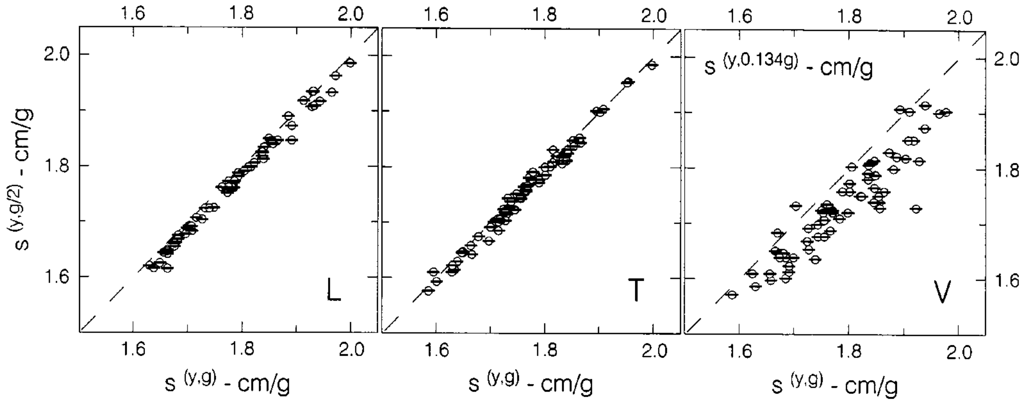

Figure 13. Comparison of two recent measurements of transducer sensitivity, both determined directly form the trace positions on the film. Values $s^{(y,g)}$ are based on trace position for 90° rotation of the instrument housing ($+g$ and $-g$ loads for the L and T transducers and $-g$ load for the V transducer). Values $s^{(y,g/2)}$ for the L and T transducers and $s^{(y,\,0{\cdot}134g)}$ for the V transducers are based on trace positions for 30° rotation of the instrument housing ($+g/2$ and $-g/2$ loads for the L and T transducers and $-0{\cdot}134g$ load for the V transducer)

non-vertical position of tilt table (δ_1 and δ_2). For typical values of $|\varphi| = 1^\circ$ and $|\delta_{1,2}| = 1^\circ$, and for φ and $\delta_{1,2}$ such that the pendulum is in its neutral position above horizontal, $s^{(y,g)}$ is overestimated by $\sim$3·5 per cent, while $s^{(y,\,0{\cdot}134g)}$ is overestimated by $\sim$0.4 per cent. For φ and $\delta_{1,2}$ such that the neutral position of the vertical pendulum is below horizontal, $s^{(y,g)}$ is underestimated by $\sim -3{\cdot}7$ per cent and $s^{(y,\,0{\cdot}134g)}$ is underestimated by $\sim -1{\cdot}4$ per cent. The distribution of misalignment angles φ, for vertical transducer (see bottom of Figure 3) then results in the observed systematic shift of measured sensitivities by about 3 per cent below the 45° line in Figure 13. Smaller values of s^{old} for V relative to L and T transducers, seen in Figure 8, is caused by the same mechanism, since s^{old} is measured directly from film by hand. The above biases are not present in s^{LSQ}, which considers all pendulum rotation angles and corrects for their effects.

Overall errors

The difference in sensitivity estimates from considering inclined ray incidence (θ_0) is systematic, and the difference from change in stiffness of the leaf spring can be related to the measured natural frequency. It is interesting then to see how the estimates of s^{LSQ} and s^{old} compare after 'removing' the parabolic trend (due to θ_0), and the estimated average changes in stiffness, measured approximately via changes in natural frequency. This is shown in Figure 14, where thus corrected s^{old} is designated by $s^{\text{old,cor}}$. To eliminate the effect of the tangent function relating the angular amplification constant (a 'real' constant for the transducer) and the sensitivity in cm/g, differences in the angular amplification are considered. Figure 14 shows $(a^{\text{old,cor}} - a^{\text{LSQ}})/a^{\text{LSQ}}$ plotted versus θ_0 for L and T transducers, and on an arbitrary axis for the V transducer ($a^{\text{old,cor}}$ is the 'adjusted' value of a^{old}). The 'adjustment' for the effect of stiffness assumes linear changes in stiffness, i.e. that the points in Figure 11 lie on the respective $y = ax + b$ lines, where $y = a^{\text{old}}/a^{w}$ and $x = (f/f^{\text{old}})^2$. It is seen that the dependence on angle θ_0 and the basis towards negative values (seen in Figure 12) are eliminated.

The remaining scatter of the points in Figure 14 should result from the other above mentioned errors, of 'random' nature. The typical bounds for such errors in estimates of sensitivity are indicated by bands (ε_1—reading accuracy, ε_2—error due to transducer misalignment, and ε_3—error due to misalignment of the instrument housing, where $\varepsilon_3 \sim \varepsilon_2$). It is seen that the scatter of the points is within the estimated overall error bounds, and that the overall accuracy of the adjusted old procedure is better than 5 per cent. It should be

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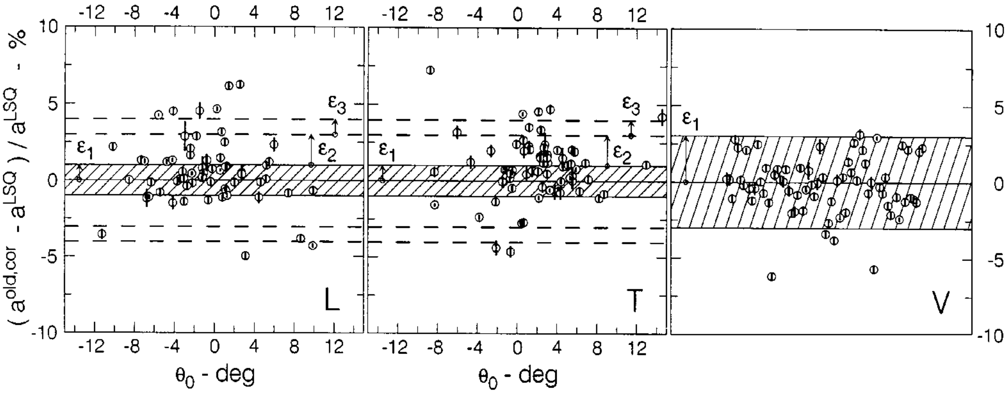

Figure 14. Percentage difference between the old ($s^{\text{old, cor}}$) and recent (s^{LSQ}, least-squares solution, considering misalignment, cross-axis sensitivity and off-normal ray incidence) sensitivities versus ray incident angle, θ_0, for the L and T transducers, and versus an arbitrary axis for the V transducer, after adjustments for the parabolic trend due to ray incident angle θ_0 and the average tend of changes in spring stiffness (ε_1—range of errors in estimation of sensitivity caused by manual reading of trace position; ε_2—range of errors caused by misalignment of transducers, and ε_3—range of errors caused by non-horizontal position of instrument housing)

noted that, in Figure 14, while 'removing' the systematic change in stiffness, additional error of random nature is introduced, which is not present in the results in Figure 10. This error comes from the errors in measurements of the natural frequency.

Testing of the accuracy of the new calibration procedure

It is difficult to evaluate the accuracy of the new calibration procedure by simple (linear) transfer-function calibration of the input–output relationships with respect to some other 'accurate' accelerometer, a common approach in many simple studies of recording instruments. This is because of coupling among the transducer sensitivity, instrument housing and transducer misalignment angles (δ_1 and δ_2, and φ and ψ), and light beam incidence angle (θ_0). There is no analogue or digital instrument which reproduces absolute acceleration of a point exactly and without some distortion. So, no single experiment (e.g. a single shaking table run) can provide all the required information without some distortion and accompanying noise.

Angles δ_1^* and δ_2^* for the 'as installed' position

In common field conditions, it is difficult (and usually not necessary) to install accelerographs so that the instrument housing is perfectly horizontal (assuming accurate alignment of the transducers with respect to the instrument housing). However, angles δ_1^* and δ_2^*, describing how much off-horizontal the instrument housing is for the 'as installed' position, can be computed by comparison of the positions of the L and T acceleration traces on the film for the 'normal' position A of a recent tilt test and for the 'as installed' position in the field. The procedure is as follows. Suppose we have obtained a tilt test result in the laboratory and we know all the instrument constants and angles δ_1 and δ_2 during the test. Then, using a fixed trace as reference, the distances of the L and T traces for the 'as installed' position from the corresponding traces for the laboratory test can be measured and converted into difference in angular deflection ($\alpha_i^* - \alpha_i$). Since the sensitivities and misalignment angles are the same, the measured differences depend only on the difference ($\delta_1^* - \delta_1$) and ($\delta_2^* - \delta_2$). Equations (28a), (29a) and (30a) from Reference 1 then imply

$$(\delta_2^* - \delta_2) = (\alpha_1^* - \alpha_1)a_1^{\alpha, w} \tag{2}$$

and

$$-(\delta_1^* - \delta_1) = (\alpha_2^* - \alpha_2)a_2^{\alpha, w} \tag{3}$$

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Computation of angles δ_1^* and δ_2^* was not performed for the Los Angeles array because (1) out aim has been only to decouple the recorded accelerations so that the motion in an orthogonal Chartesian reference frame is obtained, and (2) because the accuracy of determining the azimuth of the instrument housing is certainly not better than 1 to 2°. For special applications δ_1^* and δ_2^* can be calculated as described in this section.

SUMMARY AND CONCLUSIONS

Results were presented of recent calibration of accelerographs of the Los Angeles Strong Motion Array. An advanced procedure was used, which considers transducer misalignment and cross-axis sensitivity, as well as inclined light beam incidence onto the film.[2,3] Two of the three defined misalignment angles were estimated, φ and ψ (φ describes rotation of the transducer pendulum about the pivot axis, and ψ describes rotation of the pivot axis in the plane perpendicular to the pendulum boom). The effects of the third angle (θ) are negligible, and it was ignored.

It was found that $|\varphi| < 4°$, $|\psi| < 2°$, and $|\theta_0| < 14°$ (for the L and T transducers). The values of the misalignment angles are beyond the standard error of the calibration procedure, and are of the order of the deflection of the instrument pendulum ($1g$ acceleration along the sensitivity axis deflects the pendulum for $\sim 2°$). The sensitivity $s^{y,w} = \tan(4/a^w)$, where a^w is the amplification constant in g per radian, was found to be between 1·45 cm and 2·0 cm/g. While a^w is a constant for the transducer, the sensitivity as the ratio of the trace deflection in cm and the applied acceleration is not a constant and depends on the level of acceleration. The constant $s^{y,w}$ defined above would be the sensitivity in cm/g for $1g$ acceleration if $\theta_0 = 0$, and has been introduced for convenience as sensitivity of SMA-1 accelerographs is usually expressed in cm/g.

Angles ψ and θ_0 affect the estimate of transducer sensitivity beyond the digitization noise. Assuming $\theta_0 \neq 0$ leads to larger estimates of sensitivity. For the Los Angeles Strong Motion Array, the error in most cases is not more than 2 per cent (corresponding to $\theta_0 \sim 8°$), but for few measurements it exceeded 3 per cent. The errors in estimates of sensitivity alone will produce errors in recorded accelerations (overestimating sensitivity will lead to underestimating the ground acceleration amplitudes). Next, neglecting angle θ_0, if significantly greater than zero, will introduce further errors in converting trace deflections on the film into angular motion of the pendulum. These errors will be larger for larger recorded accelerations. Finally, there will be errors in the recorded accelerations due to cross-axis sensitivity (magnified by misalignment). These lastly mentioned errors will be the largest when the accelerations for two or for all the three recorded components of motion are large simultaneously.[2,3] These errors are important[2] for studies which rely on a single parameter of recorded acceleration, e.g. peak acceleration, and for studies in which accurate estimate of phase of motion is important[2] (e.g. system identification and differential motions). These errors can be eliminated by proper instrument calibration and instrument correction algorithms.

Comparison of the new estimates of sensitivity with those measured in 1979/80 (s^{old}) shows that, in general, the new estimates are smaller. The difference is typically not more than 5 per cent, for most cases not more than 7 per cent, but it may reach 10 per cent. Two significant contributors to the systematic difference are the changes in stiffness, and the correction for inclined light beam incidence onto the film.

A 'tilt-test' calibration run, just before installation, and one standard calibration run in the 'as installed' position of the instrument housing can give the angles δ_1^* and δ_2^* describing how much off-horizontal the perfectly aligned reference frame for the transducers is for the 'as installed' position. These angles are usually not required, but can be computed (as shown in this paper) if necessary.

This and the related paper (Reference 1) show that by detailed modeling, carefully designed calibration procedure and appropriate correction algorithms (as part of the data processing software), simple strong motion accelerographs can be calibrated accurately and band-limited representation of the actual strong earthquake motion can be produced with a high degree of accuracy. One advantage of simple analogue accelerometers, such as SMA-1, is that the parameters needed to define its mechanical model and the associated differential equations are simple, accessible, and are easy to calibrate. The misalignment and

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cross-axis sensitivity of SMA-1 transducers can be corrected with *exact* algorithms. All that is required to produce more accurate data is periodic testing and instrument calibration.

The accuracy of recording strong motion acceleration, in general, can be improved only by more precise and representative modeling of the transducers and of their recording systems. The more recently designed digital accelerometers use force-balance transducers which transmit the signal electrically, are more complex and consequently require more detailed modelling[7] and calibration. These transducers also need to be aligned and their instrument constants should be tested periodically.

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